PHY 108: Optical Physics

Solution to Homework #7

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Pulsed lasers are used for ranging (distance measurements) and for materials thickness measurements through the detection of return signals from an illuminated target. Calculate the maximum pulse length that can be utilized in the following three cases if we want distance/depth resolution that is a factor of ten better than the smallest distance/depth to be measured.

(a). A police radar that is used for measuring the distance to a speeding car at 1000 meters with a precision of 10 meters.

(b). A materials inspection system that is designed to measure variation in the thickness of sheets of the material to a precision of 1 cm.

(c). An optical tomography system that looks at the human retina to measure bleeding blood vessels behind the retina that has thickness of 50 microns.

In case (a) above, the police radar also measures the speed of the car by measuring the Doppler shift in the frequency of the return reflection from the car. Speed limit is 65 mph. The police wants to catch all speeders that violate the speed limit by 5 mph. Calculate the frequency shift measurement capability required if the laser radar is operating at wavelengths of (a) 600 nm, (b) 1.55 micrometers and (c) 10.6 micrometers.

**Solution:**

For a distance of $L$, the time it takes for a laser pulse to go a round-trip is simply $2L/c$. Therefore, to determine a distance up to a precision $\Delta L$, the maximum laser pulse length $\Delta t$ is simply:

$$\Delta t = \frac{2\Delta L}{c}. \quad (1)$$

Now if we require the distance/depth resolution is a factor of ten better, the pulse should be ten times smaller than the above value, i.e.:

$$\Delta t = \frac{1}{10} \times \frac{2\Delta L}{c} = \frac{\Delta L}{5c}. \quad (2)$$

For the three cases considered here, we have:

(a). $\Delta L = 10$ meters, $\Delta t = 6.67 \times 10^{-9}$ sec.

(b). $\Delta L = 1$ cm, $\Delta t = 6.67 \times 10^{-12}$ sec.

(c). $\Delta L = 50$ microns, $\Delta t = 3.33 \times 10^{-14}$ sec.

Now for the speed measurement, a laser of frequency $\nu_0 = c/\lambda$ is sent towards the car. Due to the Doppler effect the reflected light will have a frequency of:

$$\nu = \nu_0 \left(1 \pm \frac{v}{c}\right), \quad (3)$$
where \( v \) is the speed to be detected. The frequency shift is thus given by the expression:

\[
\Delta \nu = |\nu - \nu_0| = \frac{v}{\lambda}.
\] (4)

In order to catch all speeders that violate the speed limit by 5mph, the velocity in above expression is \( v = 65 + 5 \text{ mph} = 70 \text{ mph} = 31.1 \text{ meter/sec} \). Therefore, according to the wavelength used, the instrument should at least be able to measure a frequency shift of:

(a). \( \lambda = 600 \text{ nm}, \Delta \nu = 5.18 \times 10^7 \text{ Hz} \).

(b). \( \lambda = 1.55 \text{ micrometers}, \Delta \nu = 2.01 \times 10^6 \text{ Hz} \).

(c). \( \lambda = 10.6 \text{ micrometers}, \Delta \nu = 2.93 \times 10^6 \text{ Hz} \).